5. Indicate which of the following sentences are statements.

a. 1,024 is the smallest four-digit number that is a perfect square.

b. She is a mathematics major.

c. 128 = 26

d. *x* = 26

Answer:

1. Yes as the statement is true.
2. No she can be true or false.
3. No, mathematically false.
4. No, x can be true or false.

**Write the statements in 6–9 in symbolic form using the symbols ∼*,*∨*,* and ∧ and the indicated letters to represent component statements.**

10. Let *p* be the statement “DATAENDFLAG is off,” *q* the statement “ERROR equals 0,” and *r* the statement “SUM is less than 1,000.” Express the following sentences in symbolic notation.

1. DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.
2. DATAENDFLAG is off but ERROR is not equal to 0.
3. DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.
4. DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.
5. Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

Answer:

1. p ∧ q ∧ r
2. p ∧ ∼q
3. p ∧ (~q ∨~ r)
4. ~p ∧ q ∧ ~q
5. ~p ∨ (q ∧ r)

**Use De Morgan’s laws to write negations for the statements in 25–31.**

25. Hal is a math major and Hal’s sister is a computer science major.

Answer:

Hal is not a math major or his sister is not a computer science major.

26. Sam is an orange belt and Kate is a red belt.

Answer:

Sam is not an orange belt or Kate is not a red belt.

27. The connector is loose or the machine is unplugged.

Answer:

The connector is not loose and the machine is not unplugged.

28. The units digit of 467 is 4 or it is 6.

Answer:

The units digit of 467 is not 4 and not 6.

29. This computer program has a logical error in the first ten lines or it is being run with an incomplete data set.

Answer:

This computer program does not have a logical error in the first ten lines and is not being run with an incomplete data set.

30. The dollar is at an all-time high and the stock market is at a record low.

Answer:

The dollar is not at an all-time high or the stock market is not at a record low.

31. The train is late or my watch is fast.

Answer:

The train is not late and my watch is not fast.

**Use Theorem 2.1.1 to verify the logical equivalences in 50–54.**

**Supply a reason for each step.**

52. ∼*(p* ∨ ∼*q)* ∨ *(*∼*p* ∧ ∼*q)* ≡ ∼*p*

Answer:

≡ (~p ∧ ~(~q)) ∨ (∼p ∧ ∼q) by De Morgan’s law

≡ (~p ∧ q) ∨ (∼p ∧ ∼q) by Double negative law

≡ ~p ∧ (q ∨ ~q) by Distributive law

≡ ~p ∧ t by Negation law

~p ≡ ~p by Identity law

54. *(p* ∧ *(*∼*(*∼*p* ∨ *q)))* ∨ *(p* ∧ *q)* ≡ *p*

Answer:

≡ (p ∧ (~(~p) ∧ ~q)) ∨ (p ∧ q) by De Morgan’s law

≡ (p ∧ (p ∧ ~q)) ∨ (p ∧ q) by Double negative law

≡ p ∧ ((p ∧ ~q) ∨ q) by Distributive law

≡ p ∧ (p ∧ (~q ∨ q)) by Associative law

≡ p ∧ (p ∧ t) by Negation law

≡ p ∧ p by Identity law

p ≡ p by Idempotent law